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A Radically New Way to Tune Compound Interest and Some of Its Implications[1]

Introduction

Rising inequality in societies is being acknowledged in several sources as a major problem.[2] Inequality affects life of most people adversely, and it also harms our ability to successfully fight major challenges, including those stemming from local and global climate change.[3] Inequality has several sources, one of them being the phenomenon of compound interest.[4] In this essay I highlight a previous suggestion by von Kiedrowski and me[5] that advocated a novel way to calculate compound interest as well as a new approach for central banks to regulate interests. The common method is to tune the interest rate, but – as we shall see – there is potentially another, complementary, mechanism that would mitigate the effects of economic competition that is ultimately linked to the exponential increase of assets through compound interest. I see this suggestion as one that could help our efforts at achieving sustainable life on this planet.

Setting the stage: The appearance of subexponential growth

Compound interest implies exponential growth expressed by the formula for the calculation of continuous compounding:

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2 Bowles 2015; OECD 2015; Wilkinson–Pickett 2020.

3 Wilkinson–Pickett 2020.

4 Bowles 2015.

5 Kiedrowski–Szathmáry 2012.

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$$P(t) = P_0 e^{rt} \qquad (1)$$

Where P_0 is the principal (the initial value) and $P(t)$ is the value of the asset at time t and r is the interest rate. This formula is exactly isomorphic to unlimited exponential growth in biology and chemistry:

$$N(t) = N_0 e^{rt} \qquad (2)$$

where $N(t)$ is the number of individuals (in this case replicators) at time t and N_0 is the initial number of individuals. r in this case is the Malthus parameter affecting the vigorousness of population growth, stemming from the differential equation:

$$\frac{dN}{dt} = rN \qquad (3)$$

Population numbers go to infinity in infinite time. Note that such growth can be expressed in terms of formal chemistry as an autocatalytic process in which A is catalysing its own formation from X, while producing waste substance Y:

A + X □ 2A + Y (4)

Unlimited exponential growth requires keeping the level (concentration) of X constant, in which case its value can be absorbed into the Malthus parameter.

Exponential growth has profound consequences for selection. Assume that there is another replicator type M that grows also exponentially but with a different Malthus parameter:

$$M(t) = M_0 e^{st}, \qquad \frac{dM}{dt} = sM \qquad (5)$$

Now we can calculate the growth of the two replicators relative to each other:

$$\frac{N(t)}{M(t)} = \frac{N_0 e^{rt}}{M_0 e^{st}} = Ke^{(r-s)t} \qquad (6)$$

where K is a constant. This is also an exponential equation, and if $r > s$ then

$$\frac{N(t)}{M(t)} = \infty \qquad (7)$$

despite that fact that the number of *both* replicators also tends to infinity. The inferior competitor M is diluted out. This is the simplest demonstration of the "survival of the fittest".[6] But growth is of course not unbounded in a finite world. For the considered two competitors this can be expressed as a pair of differential equations:

$$\frac{dN}{dt} = rN - N(rN + sM)/C \tag{8.1}$$

$$\frac{dM}{dt} = mM - M(rN + sM)/C \tag{8.2}$$

where $C = N + M$ is the total population number. The equation ensures that if $N_0 + M_0 = C$ holds then C does not change (it is a "constant of motion", or "first integral" of the system), so the replicators can grow only at the expense of each other. If, as above, N is the superior competitor then with both replicators being present the system converges to the following *stable* equilibrium:

$$\hat{N} = C, \hat{M} = 0 \tag{9}$$

which ensures that (7) holds again.

Now comes the lesser known, less trivial bit. Growth equation (3) can be generalised to:

$$\frac{dN}{dt} = rN^p \tag{10}$$

where p is the *growth order* that *for exponential growth* equals exactly one. It is perhaps surprising that unless p is exactly one, the selective consequences are qualitatively different. Integration of the above rate equation yields:[7]

$$N(t) = ({N_0}^{1-p} + (1-p)rt)^{\frac{1}{1-p}} \tag{11}$$

With $p = 0$ we have linear growth and effectively no autocatalysis, so that case is of no interest here. $0 < p < 1$ is in contrast of major interest. If we do the same exercise with the relative growths as for equation (6) we see that the stable ratio remains finite. For example, if $p = ½$ then

6 Szathmáry 1991: 366–370.
7 Kiedrowski 1993: 113–146.

$$\frac{N(t)}{M(t)} = \left(\frac{r}{s}\right)^2 \quad (12)$$

implying dynamical coexistence *with selectivity* depending on the growth order *p*:[8] the superior competitor has an advantage, but cannot oust the inferior one. This conclusion remains valid for a competitive system as an analogy to equation (8). The table below shows how this stable equilibrium depends on the growth order:

p	1/2	4/5	9/10	0.99
$\hat{N}/\hat{M}$	$\left(\frac{r}{s}\right)^2$	$\left(\frac{r}{s}\right)^5$	$\left(\frac{r}{s}\right)^{10}$	$\left(\frac{r}{s}\right)^{100}$

Note that the last entry is very close to exponential growth, hence the very high selectivity. In a finite system this would already imply extinction of *M* since the minimum population number is one individual.

Subexponential growth is real in chemistry. It was discovered by von Kiedrowski in 1986 in an experimental system of the first artificial self-replicating molecule. The principle of the process is shown in Figure 1.

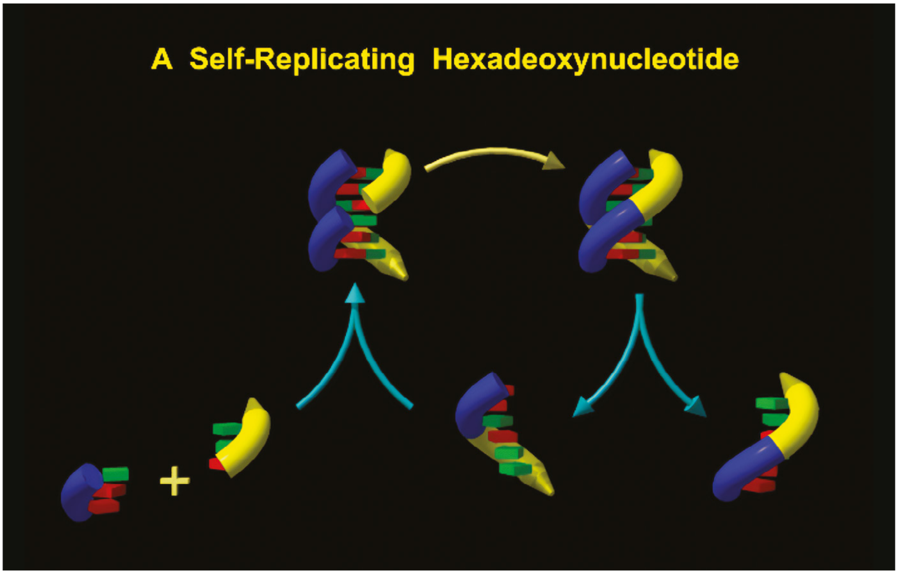

Figure 1. *The first artificial chemical self-replicator*
Source: Kiedrowski 1986: 933

[8] Kiedrowski 1993: 113–146; Szathmáry–Gladkih 1989: 55–58; Szathmáry 1991: 366–370.

The building blocks bind to the longer template. This catalyses the formation of a strong bind between the former (yellow arrow). The copy (identical to the template) can spontaneously dissociate from the template. Re-association is also possible, which inhibits grows, entailing subexponential growth.

Since then, a rich zoo of replicators has been created in labs all over the world. For us here the question is what the implications for economics might be.

The monetary growth order

The consequence for finance, economy and sustainability is that if a new method of calculating interests could be adopted, the central banks could set not only the classical interest rate (*r*) but also the monetary order (*p*). Emphatically, subexponential compound interest calculation would mitigate the rise of inequality.

It is instructive to look at examples of how assets grow in the subexponential regime (Figure 2).

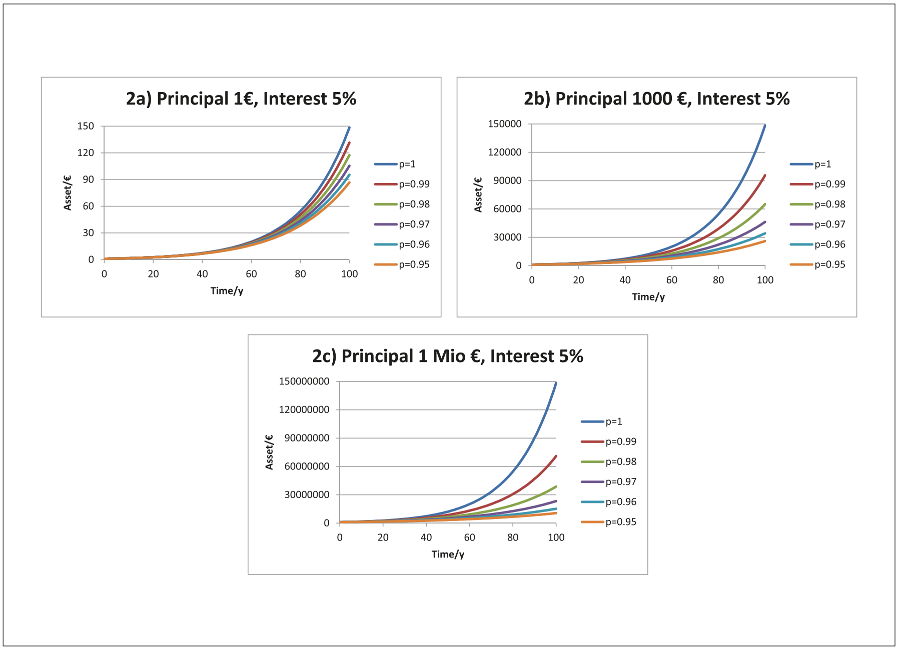

Figure 2. *Asset growth for the different principals in the exponential and subexponential domains*

Source: Kiedrowski 1986: 934

The doubling time of the principal can be calculated as follows:

$$t_2 = \frac{N_0^{1-p} 2^{1-p} - 1}{r(1-p)} \tag{13}$$

as a function of both critical parameters. Figure 3 illustrates this effect.

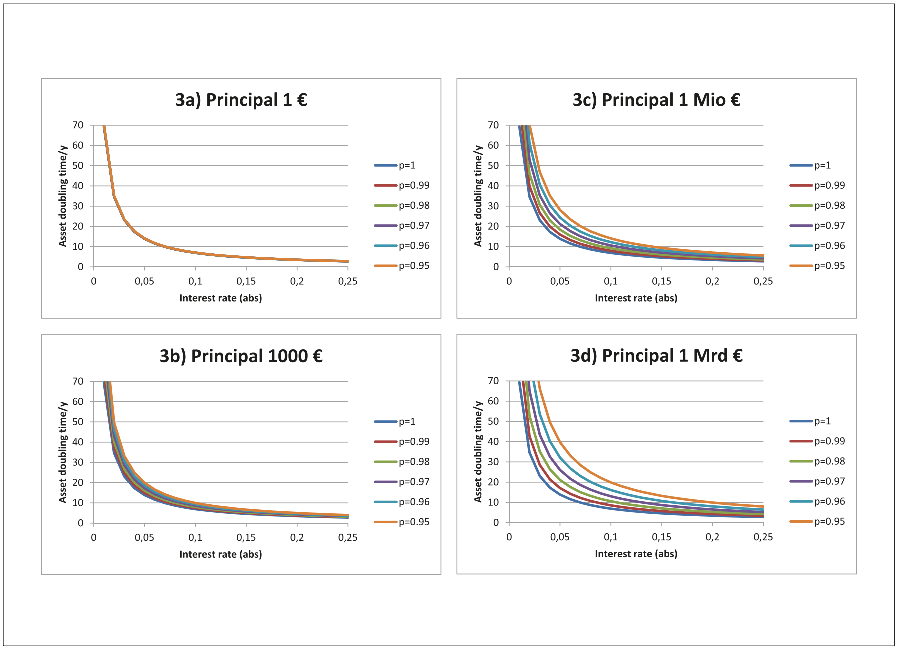

Figure 3: *Asset doubling times as a function of interest rate for different principals as a function of the monetary growth order and interest rate*
Source: Kiedrowski 1986: 934

Figure 4 illustrates the effect of competitive growth for different interest rates and growth orders. illustrates the combined effect of the parameters on the doubling time (13) for different principals.

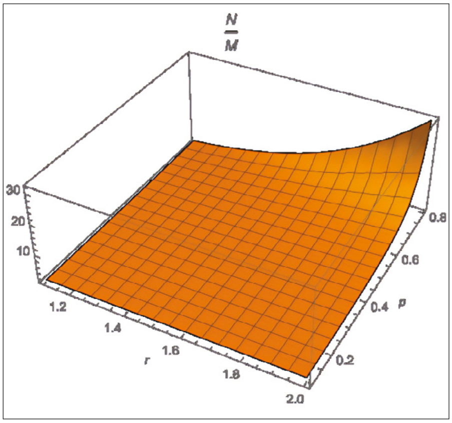

Figure 4. *The equilibrium ratio of fast (N) and slowly (M) growing assets while s = 1*
(Note the dramatic effect of the growth order on this ratio)
Source: KIEDROWSKI 1986: 935

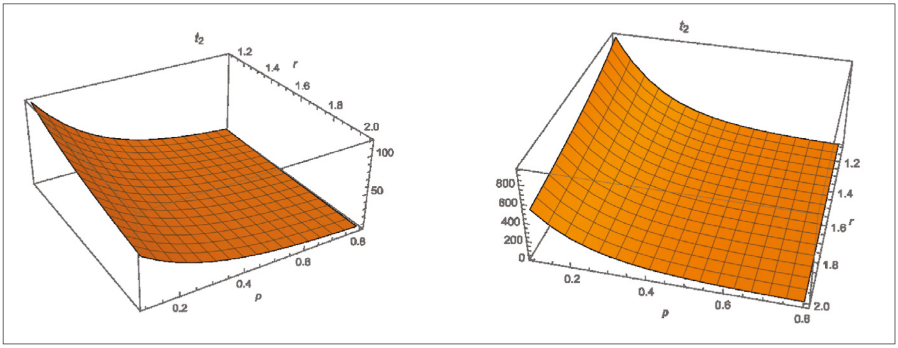

Figure 5. *Asset doubling times for = 100 and = 1000 as a function of the critical parameters*
Source: KIEDROWSKI 1986: 935

Just as we wrote 11 years ago: "The subexponential compounding mechanism is thus an effective means to stop large assets and debts creating problems of the kind we see today." There is a very important consequence of parabolic growth that is the ultimate reason of survival of everybody instead of only the fittest variant. In case of exponential growth (3) the so-called *per capita* rate of increase in constant:

$$\frac{dN}{dt}\frac{1}{N} = r \tag{14}$$

in contrast to parabolic growth (e.g. when $p = ½$) for which per capita growth rate goes to *infinity* as N decreases:

$$\frac{dN}{dt}\frac{1}{N} = r\frac{1}{N^{1/2}} \quad (15)$$

highlighting the fact that there is an *advantage of rarity* in this competitive system (Figure 6). In financial terms this translates to the statement that smaller assets grow faster. Were this effect to percolate through the economy beyond banking it would increase the establishment of startups and stimulate people to put even small savings in the bank!

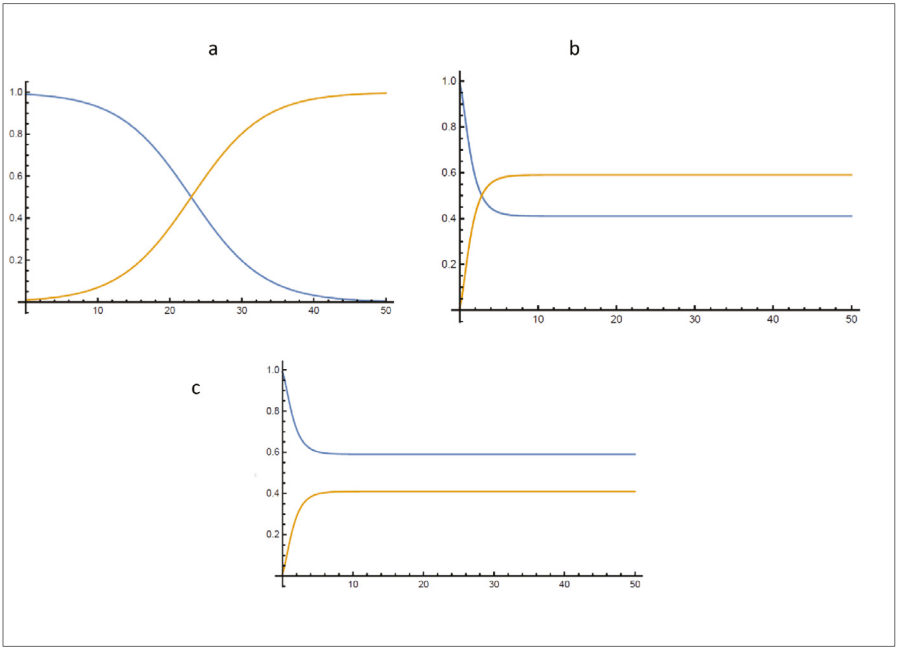

Figure 6. *Competitive growth for exponential (a) and parabolic (b, c) replicators*

Source: Kiedrowski 1986

Note: In (a) and (b) the superior species is yellow, and the inferior species is blue. Note that equilibrium in (b) and (c) is established much faster than for exponential competition (a), due to the fast invasion of the superior species. In contrast to the exponential case, even an inferior competitor can invade under parabolic competition (c). Parabolic growth leads to coexistence (b, c). Malthusian growth rates are 1.2 and 1 for the superior and the inferior competitor, respectively. Parabolic growth order is p = ½.

Discussion and outlook

In biological evolution fitness (the expected number of offspring) is maximised. This maxim is context-dependent. The environment evaluates how good are replicators as rival "hypotheses" for making a living in the particular environment. Naturally, the latter includes the other, competing and cooperating replicators. Humans have inherited aptitudes for both type of behaviour. Greed, broadly speaking, is also in our baggage. Subexponential compounding would run against it because asset growth would be self-inhibitory. The rich would get richer at diminishing rate, while subexponential compounding would favour small assets. Do we face an invincible obstacle? Maybe not.

In the first supplement to *Perpetual Peace* Kant famously wrote: "The problem of organizing a state, however hard it may seem, can be solved even for a race of devils, if only they are intelligent." The problem is: "Given a multitude of rational beings requiring universal laws for their preservation, but each of whom is secretly inclined to exempt himself from them, to establish a constitution in such a way that, although their private intentions conflict, they check each other, with the result that their public conduct is the same as if they had no such intentions.

A problem like this must be capable of solution; it does not require that we know how to attain the moral improvement of men but only that we should know the mechanism of nature in order to use it on men, organizing the conflict of the hostile intentions present in a people in such a way that they must compel themselves to submit to coercive laws. Thus a state of peace is established in which laws have force".[9]

There are indications that central banks are slowly but increasingly better aware of the issue of sustainability, including "green economy". Eleven years ago, we thought that subexponential compounding could be a major step forward in the direction of stabilising economy and society as a whole. Less inequality favours societal well-being. Naturally, open questions are a legion, such as:

- "Which macroeconomic data are to be employed to set a growth order that improves the overall performance?
- How does the monetary growth order affect the Pareto distribution, the Lorenz curve or the Gini index?
- How do the latter statistical data change with time for exponential versus non-exponential modes of monetary growth?

[9] Kant 1917: 112.

- Should a change in the monetary growth order only affect new credits or should all assets and debts, including the existing ones, continuously feel the actual growth order as a feedback?
- Is it necessary to introduce, for example, a European tax number in order to assign asset components spread over many bank accounts?
- Should the correction by the monetary growth order become an issue of merchant banking (as proposed) or taxation (as an alternative)?"[10]
- What would be the effect on other domains of the economy?
- Could shadow banking undermine central subexponential compounding?

Clearly, one and the other author of the original suggestion[11] is a chemist and a biologist, respectively, hence neither of us is a trained economist. These questions are for experts to analyse and resolve.

But there is a precedent worthy of some attention. Sir Frederick Soddy earned the Nobel Prize in chemistry in 1921. In four books, including *The Role of Money*[12] he expressed concerns about the exponential growth by compound interest, and he saw – in a Malthusian fashion – war as a means to overcome the problems caused by such growth. In Wikipedia we find the assessment "While most of his proposals – 'to abandon the gold standard, let international exchange rates float, use federal surpluses and deficits as macroeconomic policy tools that could counter cyclical trends, and establish bureaus of economic statistics (including a consumer price index) in order to facilitate this effort' – are now conventional practice, his critique of fractional-reserve banking still 'remains outside the bounds of conventional wisdom', although a recent paper by the IMF reinvigorated his proposals."

It may be time to reconsider. The future of technological civilisation may depend on this.

10 Kiedrowski–Szathmáry 2012: 12.
11 Kiedrowski–Szathmáry 2012.
12 Soddy 2003.

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